Visual Analytics of Three-dimensional Airborne LiDAR Point Clouds in Urban Regions

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Abstract

Airborne LiDAR datasets, in the form of 3-dimensional point clouds, provide geometric information, owing to their spatial nature. Their popularity as a geospatial data acquistion technique is owing to its characteristics of low noise and high point density. Our work is a step toward 3-dimensional analysis of the point clouds, where we discuss the role of visual analytics in point cloud processing. We discuss two different scenarios/applications: (a) unsupervised classification of point clouds; and (b) local geometry analysis of point clouds. For classification, we discuss both structural as well as semantic classification of the points. Structural classes are points, lines, and surfaces; and semantic classes are buildings, ground (asphalt), ground (natural), and vegetation. Owing to the nature of the object classes we focus on, the datasets of our interest pertain to urban regions, where structures belonging to these object classes are found in plenty. The local geometric descriptor is formulated by using tensor voting and gradient energy tensor, where it is comparable to the conventionally covariance matrix. Overall, our research demonstrates how adding elements of user interactivity and visualizations in a data science workflow enables users to perform first cut exploration of large scale point clouds from airborne LiDAR.

I. Introduction

Data science workflows [1] involves its four steps, namely, data preparation, analysis, reflection, and dissemination. Guo [1] has quoted [2] – *"Scientific computing is more than number crunching."* – to elucidate how data organization is a bottleneck in performing *substantive analysis*. In this paper, we revisit the processing of airborne LiDAR (Light Detection and Ranging), to which some of the findings seen in modern data science workflows can be applied. While analysis of LiDAR imagery has been matured over time, the interest in working with geometry-aware 3-dimensional point clouds is more recent [3]. The LiDAR point cloud acquisition is an outcome of advent in sensor technology used in LiDAR. Specifically in urban regions, extensive study is ongoing on building detection and reconstruction and road extraction from LiDAR imagery as well as point clouds. Rottensteiner has discussed how LiDAR point clouds are significant in combination with imagery to combat issues of occlusions, shadows, and non-detection of building smaller than $30m^2$.

Given this premise, we discuss how we have incorporated visualization in the data science workflow to process airborne LiDAR point clouds in urban regions. In two different methodologies proposed by our research group in [4] and in [5], [6], visual analytics is the key aspect of the data science workflow. Keim et al. [7] have defined *visual analytics* as an intermix of conventional data mining and interactive visualizations in a data science/analytic workflow, which unifies sense-making, inferential understanding, and decision-making support for big data. *Big data* itself is characterized by the five V's – volume, variety, veracity, velocity, and value.

Conventionally, processing of airborne LiDAR point clouds starts with identifying local neighborhood for each point. This is followed by the computation of covariance matrix $C(p) = \sum_{y \in N(p)} (y-p)^T.(y-p)$, at each point $p \in \mathbb{R}^3$. The covariance matrix is referred to as the *structure tensor*, or broadly, local geometric descriptor of the point. Most of the data mining for classification include semantic classification where features at each point are extracted and used in supervising learning techniques, such as conditional random forest classifier, support vector machines, etc. In this background, we have attempted to answer two questions –

1) How *can* we classify points if we do not have training dataset to execute supervised or semi-supervised learning algorithms?
2) How *good* is the geometric description provided by the local geometric descriptor?

In order to answer these questions we used visual analytics in combination with appropriate data modeling. Kumari and Sreevalsan-Nair [4] have used hierarchical (divisive) clustering of the point cloud to answer the issue on unsupervised classification technique. Sreevalsan-Nair and Kumari [5] have proposed a novel local geometric descriptor for LiDAR point clouds using tensor voting [8]. They have proposed processing local geometric descriptors in its form of positive semidefinite symmetric second-order tensors. Sreevalsan-Nair and Jindal [6] have further improved on the novel descriptor using gradient information. Tensor voting is a voting scheme [8] for detection and classification of feature points using proximity and continuity principles of Gestalt psychology for vote propagation.

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II. Literature Survey

In this section, we describe the relevant literature on the key topics of semantic classification as well as local geometric descriptors. Semantic classification of LiDAR data has been widely studied. We discuss some of the work which is relevant to the visual analytic framework, proposed in [4]. Further, we describe relevant work on local geometric descriptors, its tensor representations, and its uses in LiDAR research community.

Semantic (or Object-based or Contextual) Point Classification: Song et al. [9] have determined the effectiveness of using LiDAR intensity data for land-cover classification, where a uniform grid derived from point cloud is used. Chehata et al. [10] have given a classification of parameters used for semantic classification and results from using multiple classifiers using random forest classifiers. Niemeyer et al. [11] have performed supervised classification using conditional random fields (CRFs), using geometrical features and an intensity value. These results have been improved by using random forests with the CRFs in [12]. Niemeyer et al. [13] have proposed inclusion of context of spatial locality, as an additional cue to the supervised classification.

Other Similar Methodologies: Ramiya et al. [14] have used curvature and colorimetric distance for segmenting colored LiDAR data. Other unsupervised classification techniques, such as [15], [16], exist, which use density-based clustering and graph-cut based methods. We have found applications of interactive agglomerative clustering, which is bottom-up approach of merging clusters. Preiner et al. [17] have used hierarchical agglomerative Expectation Maximization clustering for surface reconstruction from point cloud data, unlike the divisive clustering method used in [4].

Covariance Analysis: Covariance analysis of local neighborhoods based on centroid is a robust method for normal estimation [18], but not necessarily for finding the shape of the neighborhoods. Tombari et al. [19] have made the argument of lack of repeatability of sign of a local reference frame when using the covariance matrix, and have proposed a weighted covariance matrix based on the point itself instead of the centroid, for surface matching. Local tensor-based techniques are a tradeoff between computational complexity and accuracy in feature detection; e.g. use of tensor voting [20], [21] for feature classification.

Tensor Representation of Local Geometric Descriptor: Knutsson [22] has defined a structure tensor as a tensor definition for based on differentiation of functions. Structure tensor has been used as a descriptor in 3-dimensional space. Knutsson et al. [23] have discussed different descriptors used for images, with a potential for extension to 3D point clouds.

Structural Classification: Structural (or geometric or feature) classification of point clouds has been less explored [20], [24]. Structural classification is implicitly used in semantic classification through the use of eigenvalue-based features obtained from the local geometric descriptor, such as covariance matrix in LiDAR point clouds [25], [4].

Multi-scale Classification: Pauly et al. [26] have proposed the use of multi-scale surface variation, estimated using covariance matrix of local neighborhood. There, surface variation at a user-defined scale gives feature weights, which on appropriate thresholding gives features. Keller et al. [27] have used a similar multi-scale approach, for LiDAR point clouds, in determining feature weights from covariance matrix of local neighborhoods. However, the difference between the methods in [26] and [27] is that a single adaptive scale and averages across multiple scales have been used, respectively. Algorithms for finding optimal neighborhood size or scale has been of interest to the LiDAR community [25], [28], [29]. Blomley et al. [30] have used multi-scale approach using shape distribution features for point classification, as opposed to covariance features, proposed by Keller et al. [27]. Park et al. [20] have used tensor voting and surface variation to classify and detect line features in point clouds, where th surface variation function is computed using a multi-scale method. An approach based on anisotropic diffusion is used in [21], where anisotropic diffusion is performed after tensor voting for feature classification and extraction in polygonal mesh data, and subsequent mesh segmentation.

III. Augmented Semantic Classification

Semantic classification of LiDAR point clouds has been extensively studied. As discussed in Section II, many of the recent methods rely on supervised learning methods [13], [31], [32]. Supervised learning methods require training datasets, which are conventionally generated by a domain expert, data from the site measured using complementary processes, or other publicly available alternative (albeit partial) sources, e.g. Openstreetmap [33] and Google Earth [1]. In order to have a facility to visually explore new airborne LiDAR datasets with a preliminary semantic classification, Kumari and Sreevalsan-Nair [4] have proposed a visual analytics framework using unsupervised learning for semantic classification. The novelties of the method proposed in [4] are: (a) the concept of *augmented semantic classification*, (b) interactive setting of parameters in the visualization of the tree data structure, representing hierarchical clustering method, and (c) the use of visualizations to guide setting the features to be considered for clustering at each level of hierarchy. They have demonstrated a prototypical tool implementing

[1]Google Earth at https://www.google.com/earth/ Last retrieved on January 09, 2018.

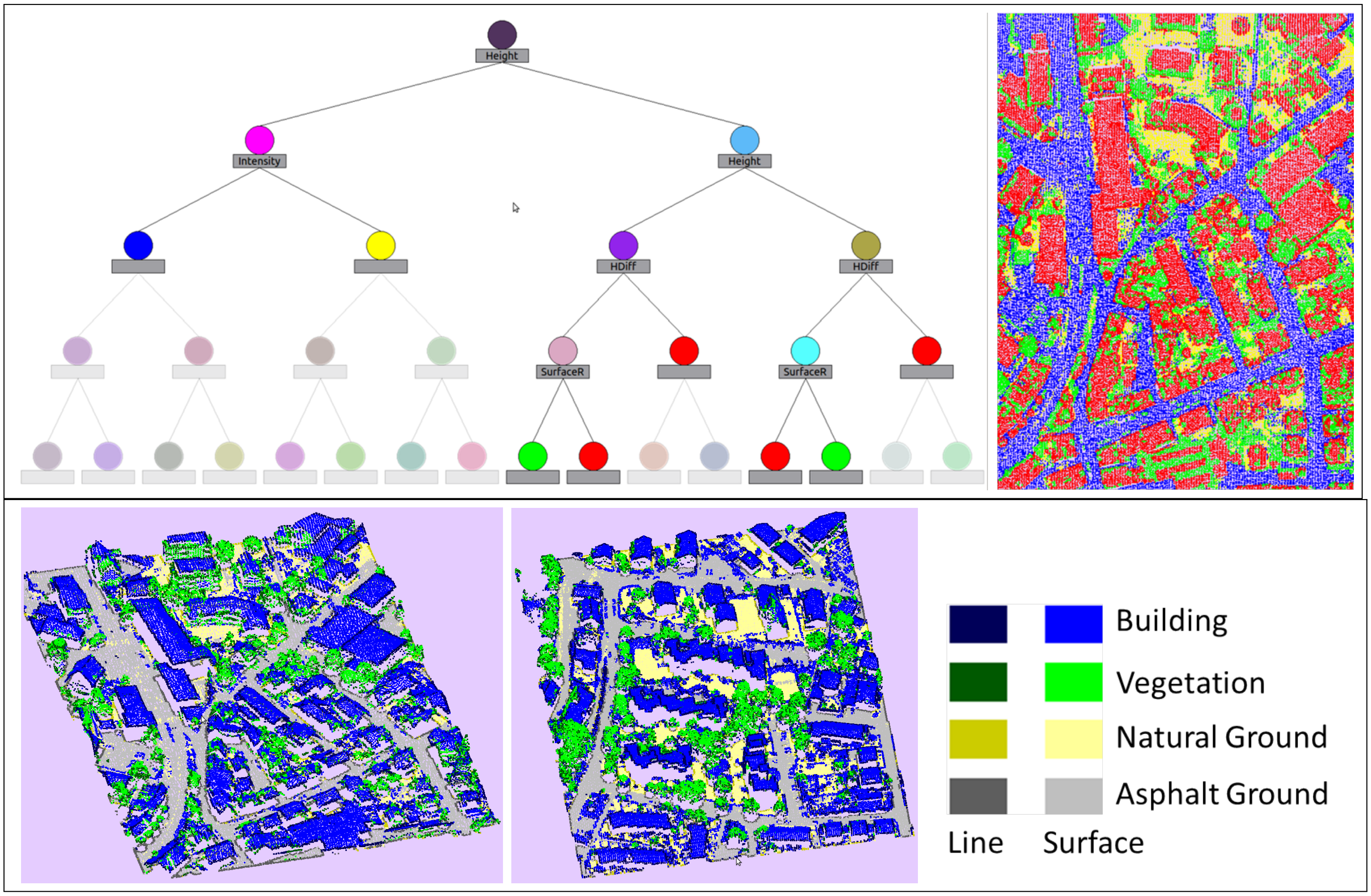

Fig. 1: (Top) The graphical representation of hierarchical divisive clustering of airborne LiDAR point clouds, uses tree data structure (left). Theactive leaf nodes of the tree, which are not dimmed/transparent, belong to four different object classes. The points of a class can belong to multiple leaf nodes, e.g. red nodes correspond to building. The point rendering shows the state of the active leaf nodes of the tree (right). (Bottom) The results of augmented semantic classification of Area-1 (left) and Area-2 (middle) of Vaihingen dataset, as per the legend showing tuple of labels in a matrix (right). The bottom image hase been modified from an image in [4].

the proposed visual analytics method, referred to as the *tree visualizer* (Figure 1). The experiments have been done on the Vaihingen dataset [34], provided by the German Society for Photogrammetry, Remote Sensing and Geoinformation (DGPF)[2]

Unsupervised Semantic Classification Using Tree Visualizer: With the recent success seen with supervised learning, one cannot discount the value of such a class of methods for semantic classification. However, as discussed earlier, the current state of the art methods develop training models separately for different datasets. The reason for this could be that this work is fairly recent, and additionally, it could take several experiments to derive a generic training model(s) owing to inherent differences in built environments worldwide. Hence, for first-cut and quick exploration of new datasets, Kumari and Sreevalsan-Nair [4] have proposed the use of unsupervised methods, in an adaptive manner to accommodate the high variability in environmental data in airborne LiDAR point clouds. The idea is to provide several iterations of classifications and adaptiveness of the method is provided by the choice of parameters or features to be used for an iteration of classification in different regions. Given these requirements, hierarchical (divisive) clustering is the most appropriate unsupervised method that can be used for unsupervised classification problem in this case.

Kumari and Sreevalsan-Nair further show the use of hierarchical EM clustering specifically. In this proposed method, the classification is agnostic of the spatial locality. Hence, eventually the spatial locality information is added in a post-processing step. This step involves a region-growing algorithm [35], which is used for correcting the labels of points based on the majority *vote* of the label of the points in its local neighborhood.

Choice of Feature Vector for Hierarchical Clustering: The supervised learning methods for semantic classification have shown the set of features that are required in the feature vector [10]. Kumari and Sreevalsan-Nair have chosen the commonly

[2]DGPF at http://www.ifp.uni-stuttgart.de/dgpf/DKEP-Allg.html

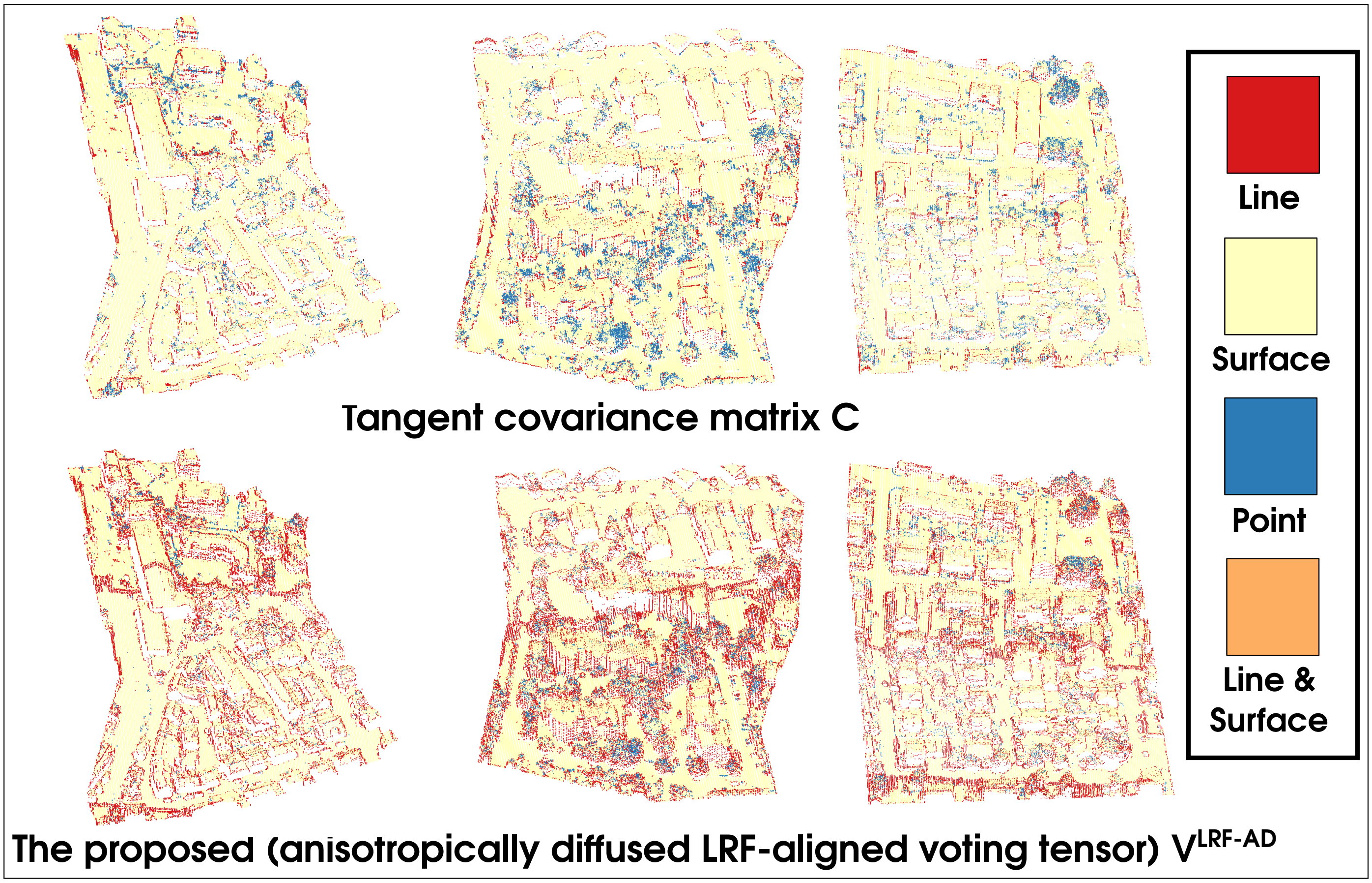

Fig. 2: The point rendering of (left-to-right) Area-1, Area-2, and Area-3 of Vaihingen benchmark dataset show differences in structural classification based on the use of conventional (tangent) covariance matrix (top) with respect to the anisotropically diffused tensor voting based local geometric descriptor (bottom). This image is a modified version of an image in [5].

used parameters or features, such as height, intensity, height variance, and height range. There are a set of features which are derived from the covariance matrix of the local neighborhood of each point. The covariance matrix is the local geometric descriptor [11], which we discuss further in more detail, in Section IV. The colormaps or heat maps of the different features are then used for providing colors to the points with corresponding feature values. These visualizations allow the user to make decisions on which parameter, according to their visual perception, gives the best binary *clustering* of the concerned subset of points. The process entails the entire point cloud being clustered in the leaf nodes of the clustering hierarchy. Since the number of leaf nodes in a binary tree, which need not be balanced, exceeds the number of semantic classes, some of the clusters are appropriately merged to give the exact number of semantic classes.

The Proposed Augmented Semantic Classification: There are two types of classifications possible for the airborne LiDAR cloud, namely, geometric (or structural) and object-based (or semantic). Semantic classification [13], [31], [32] is widely studied than structural classification [27], [36]. Structural classification labels the points as belonging to line-, surface-, or (critical) point-type features. The latter includes features like junctions. Conventionally, structural classification is obtained using the eigenvalue analysis of the local geometric descriptors. Thus, structural classification is used as an intermediate step in semantic classification, e.g. linear and areal anisotropies, which are indicative of structural classification. Kumari and Sreevalsan-Nair [4] have proposed to preserve the structural classification by introducing a tuple of labels for each point in the point cloud, instead of a singleton label. The tuple of labels includes both structural and semantic labels. The use of tuple of labels for each point in the point cloud has been called *augmented semantic classification*. The benefit of the augmented classification [4] has been in the improved rendering, as shown in Figure 1, where the line-type features make the rendering of the point cloud sharper.

Results: A domain expert user study was done on the results of unsupervised semantic classification of Vaihingen benchmark dataset, in the absence of ground truth [4]. Visually, the domain expert has determined an overall accuracy of 80-85% in the classification. The point rendering of the LiDAR dataset using color scheme based on augmented semantic classification is visibly sharper than that of semantic classification, as shown in Figure 1.

IV. Local Geometric Descriptors

The decisive role of local geometric descriptors in both structural as well as semantic classifications begs further in-depth research on them. This area of research has been under-represented in the LiDAR community, unlike the computer graphics or geometric modeling communities [37]. Hence, we showcase work in deriving new local geometric descriptors in airborne LiDAR point clouds, and its comparative analysis in [5], [6]. While covariance matrix is used ubiquituously for classification, it has been found to not identify the sharp line-type features, e.g. gable line in the roofs. Hence, the motivation is to identify a local geometric descriptor that behaves similar to the covariance matrix C, and at the same time, highlights sharp features.

Definition: A *local geometric descriptor* of a point, p, is defined as the data entity that captures the shape of the local neighborhood of a point p, $\mathscr{N}(p)$. This is a descriptor, because it defines the type of geometric feature of which the point itself belongs to. Say, a point with cylindrical neighborhood belongs to a line-type feature; with disc-shaped neighborhood, to a surface-type feature; and with spherical neighborhood, to a critical point-type feature [27]. Thus, we can see how local geometric descriptors are significant for structural classification. In order to determine the shape of the local neighborhood, one has to find the eigenvalue decomposition of the descriptor. Thus, the eigenvalues of the local geometric descriptor are required for both structural as well as semantic classification. The local neighborhood itself can be either defined in terms of points within a specific Euclidean distance, which gives a spherical neighborhood; or in terms of the number of nearest points, e.g. k-nearest neighbors, for $k \in \mathbb{Z}_{>0}$.

Tensor Voting for Local Geometric Descriptor: Inspired by the work by Wang et al. [21] on the use of tensor voting [8] for extracting sharp (line) features in triangular meshes, Sreevalsan-Nair and Kumari [5] have used tensor voting to define the local geometric descriptor for LiDAR point clouds. On comparative analysis of the tensor voting descriptor, V, computed for unoriented points as discussed in [20], with the conventionally used covariance matrix, C. It has been determined that C and V are both positive semi-definite and symmetric second-order tensors. One of the differences between C and V is that they are generated in the tangent and normal space, respectively. Hence, this property of same type makes the local geometric descriptors comparable and substitutable.

Visual Analytics: Visualization has been used for qualitatively comparing different descriptors in [5]. Two channels are required for encoding the eigenvalues and eigenvectors, namely color and geometry. The eigenvalues gives the structural classification, and the eigenvectors give the orientation of the tensor, which is a representative of the local geometric descriptor. Colormap chosen has been based on the saliency maps of the point. The saliency map gives the likelihood of the point to belong to any of the three structural classes. For geometry channel, both point rendering (Figure 2) as well as superquadric tensor glyphs [38]. The shape of the superquadric tensor glyphs and the color encoding based on saliency map redundantly give the perception of the structural classification of the points [5].

Anisotropic Diffusion and Local Reference Frame Alignment: The qualitative comparison of the two local geometric descriptors, C and V, has revealed that the structural classification given by them are not equivalent. This difference has been attributed to the generation of C in the tangent space as opposed to V in normal space. At the same time, we see that Wang et al. have used *anisotropic diffusion*, using Gaussian weights and reciprocal to *flip* the sorted order of eigenvalues. Thus, we see that anisotropic diffusion performs two actions: (a) "corrects" the structural classification given by V to be similar to C, and (b) slows down diffusion across sharp features and speeds it up along sharp features. The overall outcome of the anisotropically diffused tensor voting based local geometric descriptor, V_{AD}.

Multi-scale Aggregation: The LiDAR point cloud analysis is generally performed on multiple scales. Multiple scales are assigned based on the definition of the local neighborhood. There are two ways of combining the analysis across multiple scales. One is to use optimal scale, based on optimizing a property, e.g. entropy [39] or aggregation using saliency maps [27].

Further, Sreevalsan-Nair and Kumari have found that the orientations of the local geometric descriptors, based on their eigenvectors need to be aligned in order to compare the descriptors effectively. Hence, they align the eigenvectors, thus, aligning the local reference frame, for comparing two different local geometric descriptors. The descriptors with local reference frames aligned are referred to as C^{LRF} and V^{LRF-AD}, respectively. Sreevalsan-Nair and Kumari have observed that the use of V^{LRF-AD} enables identification of gable roofs, by strengthening an otherwise weak line-type feature. However, this method reduces the number of points belonging to critical point type features. The presence of both line- and point-type features are essential for effective geometric reconstruction.

Improvement using Gradient Energy Tensor: To improve the identification of point-type features, Sreevalsan-Nair and Jindal [6] have proposed the use of gradients. The gradients, from first- to third-order derivatives, are computed for each point to give the gradient-energy tensor (GET) [40]. The use of GET has been proposed owing to two reasons: (a) it detects points of interest, which are essentially junction points, and (b) in order to find derivatives, geometry of the point cloud must be described as a function or a map. This is possible for LiDAR point clouds, as height is a function of (x, y), thus exploiting the 2.5-dimensional nature of the point cloud data.

The usage of GET tensor, however, requires post-processing, as it is a 2D tensor (which need not exist as positive semi-definite second-order tensor, but can be used as positive semi-definite second-order tensor), which is to be used with a 3D tensor, V^{LRF-AD}. Exclusively for the points of interest detected by the 2D tensor, a mapping of the eigenvalues and eigenvectors between the 2D and 3D tensors is performed to complete the correction. Testing this new tensor on the Vaihingen dataset, shows improved results in detecting point-type features.

Results: For Area-1 of Vaihingen dataset, the local geometric descriptors classify the following percentages of points as (line, surface, point) type features: covariance matrix C gives (24%, 60%, 16%); the proposed anisotropically diffused tensor voting based local geometric descriptor, without GET correction gives (29%, 65%, 6%) and with GET correction gives (27%, 63%, 10%). We observe that, as per design, V^{LRF-AD} identifies more line-type features than conventionally used C, as shown by the higher red tinge in datasets with V^{LRF-AD} in Figure 2. We also see that the point-type features reduce drastically. Further, correction using GET tensor improves the percentage of point-type features.

V. The Role of Visual Analytics in Point Cloud Processing

In Sections III and IV, we have seen two different applications of visual analytics in processing airborne LiDAR point clouds. One has been in using visualizations to guide unsupervised (semantic) classification [4]; and the other has been in using the same in comparing local geometric descriptors [5], [6], which can potentially impact both structural, and, subsequently, semantic classifications. These works go to show that data analytics can be conducted in an iterative and interactive manner, by introducing the feedback loop that is present in visual analytics workflow. These works also go on to show how using visual analytics can bring further insights to the dataset. Using visualization for exploration allows the viewer to adapt analytic processes for achieving better insights to the data. A visual analytic framework, as proposed in [4] allows the user to flexibly move across different actions in a data science workflow.

These works have the potential to drive further research. The tree visualizer based unsupervised classification is yet to be analyzed in terms of accuracy as well as adaptability. A richer analysis may be done based on cluster shapes which the human-in-the-loop can perceive before choosing the parameters for clustering. Similarly, the use of new local geometric descriptors, such as the one based on tensor voting, is yet to be tested for both classification as well as geometric reconstruction.

LiDAR point clouds can be considered as **spatial big data** [41] given the *volume* and *variety* in the data. In a specific instance, Cugler et al. have discussed how spatial big data may use computational models to generate hypotheses, which can be further used for answering questions for which the hypothesis applies. In [4], [5], [6], we observe that different visualizations have been used to create hypotheses for questions on which features reduce the uncertainty of semantic classification of points, can intensity be used as a reliable feature for classification in a given dataset, and which points are the best to be used for extracting footprints of buildings. While the results of visualization are qualitative, the outcomes become quantitative as we move through the data science workflow. Nonetheless, the quality of the outcomes is limited by the the ability of the viewer to make *relevant* sense of the data.

In conventional LiDAR point cloud processing, visualizations are limited to rendering points based on height or semantic class. In this paper, we have discussed two different applications where visualizations are used for visual exploration of datasets. This is particularly useful in order to use the knowledge discovered by perception for further analysis of the data. This is a thread which can be further pursued for research on what are different data models, like the local geometric descriptors, which can have two-fold benefits, namely: (a) the data model influences appropriate visual representation of the dataset; and (b) the data model can give insights of the data or enable big data analytics, in its own right. By data model, we refer to appropriate representations of the whole or partial data, without distorting the overall understanding of the data. Appropriate choice of data structures helps in determining apt data models. An example of a data model is the feature graph by Keller et al. [27], which was instrumental in reducing the point cloud without losing significant features. Representing each point in the point cloud as a second order tensor using local geometric descriptor is another example of data model.

In order to tackle the aspect of *volume* of the LiDAR point clouds, spatial data structures such as octree and kd-tree, are used for storing the data and for speeding up search operations to compute local neighborhood in [4], [5], [6]. In addition to this, the tree visualizer in [4] iteratively subdivide the point cloud as the hierarchical EM clustering is executed at each node of the clustering tree. Thus, the leaf nodes of the tree visualizer correspond to subset of points with a specific label, and by design, a point belonging to a leaf node, belongs to its parents up all the levels till the root node. The user can choose an action of binary clustering at a leaf node, using the user-defined parameter(s). Thus, the tree visualizer is designed for the user to interactively change the choice of feature/parameter for clustering, but this action does not process the entire point cloud unless the root node is activated for modification. Hence, tree visualizer implements a *divide-and-conquer* approach.

VI. Conclusions

In this paper, we have summarized the combined significance of the works done in our research group in [4], [5], [6] in visual analytics of airborne LiDAR point clouds. We have discussed how visualizations can guide decision making for improving unsupervised classification in [4]. The new local geometric descriptor introduced in [5] using tensor voting, and its improvement using gradients [6] show how visualization can enable the *substitutability* of local geometric descriptors. Local geometric descriptors are conventionally used for both structural as well as semantic classifications. We have shown how the research on processing point clouds acquired by airborne LiDAR is a topic considered in spatial big data analytics. We have briefly described how visualizations can be used beyond summarizations to make sense of the data. This is just the tip of the iceberg on how the use of visualizations of the 3-dimensional LiDAR can usher in multidisciplinary approaches for data analytics.

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