BOOLEAN CHITRAKAVYA

G N S Prasanna, Rajesh Kumar, Omanand Jha Vatsa, Gaurav Srivastav

Abstract— The research work presents the transformation of an integer represented in the binary field of ones and zeros to an other integer by arranging the bits of the number on a m x n matrix and performing a knight's tour on it and finding the numbers which when transformed produce the same number, i.e., itself. The scheme has been generalized for an integer of m x n bits arranged on a m x n matrix and performing all the possible solution for knight's tour for a m x n matrix.

Index Terms—Chitrakavya, Chitrakavya Number, Fixed Points, Chitrakavya Order, Regular Expressions

I. INTRODUCTION

Chitrakavyas are the kavyas or the slokas (poems) embedded in a picture found in Indian literature prominently in Sanskrit and other Indian languages. Some of the pictures included the solution of knight's tour which was evident from the fact that the aksharas (alphabets in devanagri script) of one of the slokas which was arranged on a matrix and when read sequentially made a meaningful sloka. The other sloka was formed by only using the letters of the previous sloka and when their position was observed on the matrix, the second sloka was found to be made solely from the letters of the previous sloka as well as their position in the original matrix provided a solution to the knight's tour problem.

Not only two meaningful and syntactically correct slokas are generated , but also they are almost of the same meaning.

This concept from Indian literature leads to the idea that the same concept could as well be applied to any binary number. The idea is to represent a decimal number into its binary equivalent, arrange the bits on a matrix, perform a knight's tour on that, and if the number generated after performing the knight's tour is same as the original number we have found a Chitrakavya number (i.e. a number which when transformed under knight's tour generate the same number).

The fundamental idea involved is the generation of a number transforming the original number by application of the knight's tour over a binary field of zeros and ones instead of the n-ary field of alphabets of the devanagri script. And instead of generating meaningful and same slokas, numbers are generated which are same as the original numbers on which knight's tour transformation is done.

The research work in this field concentrated on many important aspects. One of the aspects is to find the Chitrakavyas number represented by any number of bits and not limited by the hardware support of the computer. Typically representing integers whose number of bits exceeded 32 bits was not possible in the normal desktop systems. The other aspect relates with time efficient mechanism to generate all Chitrakavya numbers represented by n bits arranged over a matrix of size a x b = n. The generation of Chitrakavya numbers requires all possible solution for Knight's tour for given a x b matrix and application of all these possible paths for all possible numbers of n = a x b bits . Also the generation of all possible solution for a knight's tour for a x b size matrix required an efficient algorithm.

We devised our own algorithm to find the entire possible path for a knight's tour over matrix of size m x n where m and n are arbitrary integers. Having found all the possible paths for a knight's tour, the task was to generate all possible numbers represented by m x n bits , perform knight's tour over each one of them and to determine whether on performing a specific knight's tour the resulting number was same as the original one or not. If the transformed number was same as the original number is named as a "Chitrakavya Number" for that specific knight's tour.

The method of finding a "Chitrakavya Number" is also enhanced such that if on first transformation the resulting number is not same as that of the original one, the resulting number is now taken as a temporary number and the same Knight's tour was performed on this temporary number. This process continues until the resulting number is not equal to the original number or if the number of iterations is m x n -1 for an m x n bit number transformed by a knight's tour of m x n matrix. The number of iterations I at which the transformed number becomes equal to original number has been termed as "Chitrakavya Order". A valid "Chitrakavya Number" has a "Chitrakavya Order" greater than zero and less than number of bits m x n required for its representation in binary.

In section II, the way in which an integer is represented on an m x n matrix is discussed. Also, the way in which a knight's tour is performed on these numbers is shown. In section III and IV, the method to find the "Chitrakavya Number" has been discussed and the method to determine the "Chitrakavya Order" has been dealt with. Various results associated with Boolean Chitrakavyas has been presented in Section V. Section VI deals with multipath chitrakavya numbers. Section VII deals with permutations in regular expressions to obtain syntactically correct expressions.

II. REPRESENTATION OF AN INTEGER IN M X N BITS AND IT'S TRANSFORMATION BY A KNIGHT'S TOUR FOR A CHESS BOARD OF SIZE M X N.

As we know that the knight's tour is possible only for a matrix of a size m x n where $m \geq 4$ and $n \geq 3$ or $m \geq 3$ and $n \geq 4$. So we will transform any integer which can be represented by 12 bits through a Knight's Tour.

Let us take a matrix having 4 rows and 3 columns. If we take any integer , say, 256, its binary representation will be given by 000100000000 , i.e. , we have 9^{th} place set to 1. Now, if we arrange it over having 4 rows and 3 columns, the least significant bit will go on row = 0, column=0, the next bit goes to row = 0 and column =1, the next bit goes to row = 0, column =2, and we thus go on placing the bits and the most significant bit occupies row = 3 and column = 2. In this way we represent the bits of an integer on the matrix.

Having arranged the bits on the matrix, we now take any solution of the Knights tour for a matrix of size 4 x 3. First of all we mark the first row and first column as 0, first row and second column as 1, first row and third column as 2 and we go on proceeding like this and mark the last row and last column as 11. These are essentially the positions in the matrix and also they signify the bit number of the original integer placed at these positions.

So we have the following schema
initially: position 0 1 2 3 4 5 6 7 8 9 10
11 bits lsb to msb 0 0 0 0 0 0 0 0 1 0 0 0

Lets take a solution for Knights Tour for a matrix having 4 rows and 3 columns.
The sequence of Knights Tour is given
as: Knight's Tour 0 7 2 3 10 5 6 11 4 9 8 1

Looking at this tour, we shift the bits in the fashion as illustrated:
Put the corresponding bits of the original number according to Knight's Tour.

The Knight's Tour is	0	7	2	3	10	5	6	11	4	9	8	1
The rearranged bits	0	0	0	0	0	0	0	0	0	0	1	0
The actual position	0	1	2	3	4	5	6	7	8	9	10	11

The transformed number in binary is 010000000000 which is decimal equivalent for 1024.

III. METHOD TO FIND THE CHITRAKAVYA NUMBER OF ORDER 1

If we have a of size M x N we represent the position at ith row and jth column by (N x i)+j = k, say. Now the bit at position k is represented by a variable X(k), $0 \leq k \leq MxN-1$.

Now corresponding to a given Knight's tour we can form a set of linear equations. If the j th move by the knight goes to position z, and if we represent the bit at position z of the as X(z) , then X(z) must be equal to X(j) if the number is a Chitrakavya number for all $0 \leq j \leq n-1$.

For example let us take a of size 4 x 3and represent it as shown below:

row ↓col→			
	X(0)	X(1)	X(2)
	X(3)	X(4)	X(5)
	X(6)	X(7)	X(8)
	X(9)	X(10)	X(11)

Let us take a Knight's tour for a of size 4 x3. One of the possible sequence of moves is
0 7 2 3 10 5 6 1 8 9 4 11
The obtained by transformation by Knight's tour is :

row ↓col→			
	X(0)	X(7)	X(2)
	X(3)	X(10)	X(5)
	X(6)	X(1)	X(8)
	X(9)	X(4)	X(11)

To find a chitrakavya number, we essentially form a set of linear equations as shown below:
X(0) = X(0);
X(1) = X(7);
X(2) = X(2);
X(3) = X(3);
X(4) = X(10);
X(5) = X(5);
X(6) = X(6);
X(7) = X(1);
X(8) = X(8);
X(9) = X(9);
X(10) = X(4);
X(11) = X(11);
On removing redundant equations, we get
X(0) = X(0);
X(1) = X(7);
X(2) = X(2);
X(3) = X(3);
X(4) = X(10);
X(5) = X(5);
X(6) = X(6);
X(8) = X(8);
X(9) = X(9);
X(11) = X(11);

For this Knight's tour we observe fixed points 0,2,3,5,6,8,9,11 as these are the positions which match in the original and the transformed .

The number of linearly independent equations determine the number of chitrakavya numbers we can obtain for the particular Knight's tour which has been used to transform the original of bits. If there are m linearly independent equations, we will get 2^m chitrakavya number for that Knight's Tour. Since 1≤m ≤ n where n = number of bits , we will have at least 2 chitrakavya numbers and at most 2^n chitrakavya numbers for a Knight's Tour. When we have only one equation i.e., m=1, the chitrakavya numbers are 0 and 2^n-1 where n is the number of bits. When m=n, all the number i.e., from 0 to 2^n – 1 are chitrakavya numbers.

To generate all the chitrakavya numbers, we take m variables Y(i), 0≤i≤m-1, and m = number of linearly independent equations obtained after the transform. Thus we take variables
Y(0), Y(1),Y(2),……., Y(m-1).

For the above example, we will have Y(0), Y(1), Y(2), Y(3), Y(4), Y(5), Y(6), Y(7), Y(8), Y(9) since we have only 10 linearly independent equations from 12 equations and modify the simultaneous equations as shown below:

Y(0)=X(0) = X(0);
Y(1)=X(1) = X(7);
Y(2)=X(2) = X(2);
Y(3)=X(3) = X(3);
Y(4)=X(4) = X(10);
Y(5)=X(5) = X(5);
Y(6)=X(6) = X(6);
Y(7)=X(8) = X(8);
Y(8)=X(9) = X(9);
Y(9)=X(11) = X(11);

To generate all the chitrakavya numbers of order 1, we generate all possible numbers of m bits i.e. 0 to 2^m-1 where m is the number of independent linear equations obtained after the transform. The m bits of the resulting numbers are then assigned to Y(i) as follows:
Y(i) = ith bit of the number

Having assigned all the bits, we can now easily determine the chitrakavya numbers from the corresponding modified equations. For the above example, let us take a 10 bit number, say 256, i.e, 0100000000. The 12 bit chitrakavya number corresponding to this will be 001000000000 i.e., 512 is a chitrakavya number of order 1 for the particular Knight's Tour which has been used to transform the .

Following this procedure we can obtain all chitrakavya numbers for that particular Knight's tour. Also for any Knight's tour, 0 and the number represented by 1s at all the bit position are always chitrakavya numbers.

The method discussed above gives all chitrakavya numbers for all Knights Tour over a of size M x N which are of order 1. With slight modifications in this scheme, we can generate chitrakavya of order K provided K ≤ M x N which is discussed in the next section.

IV METHOD TO FIND THE CHITRAKAVYA NUMBER OF ORDER K

Let us take the example discussed in the previous section. The original mateix was given by

row ↓col→			
	X(0)	X(1)	X(2)
	X(3)	X(4)	X(5)
	X(6)	X(7)	X(8)
	X(9)	X(10)	X(11)

and the Knights Tour was
0 7 2 3 10 5 6 1 8 9 4 11

For order 1 we transform it one time using the Knight's Tour and the resultant was

row ↓col→			
	X(0)	X(7)	X(2)
	X(3)	X(10)	X(5)
	X(6)	X(1)	X(8)
	X(9)	X(4)	X(11)

Let this be given as

row ↓col→			
	Z(0)	Z(1)	Z(2)
	Z(3)	Z(4)	Z(5)
	Z(6)	Z(7)	Z(8)
	Z(9)	Z(10)	Z(11)

where

Z(0)=X(0)
Z(1)=X(7)
Z(2)=X(2)
Z(3)=X(3)
Z(4)=X(10)
Z(5)=X(5)
Z(6)=X(6)
Z(7)=X(1)
Z(8)=X(8)
Z(9)=X(9)
Z(10)=X(4)
Z(11)=X(11)

Let us denote this by Z. Now we take the Knight's Tour transform of Z. This will be

row ↓col→			
	Z(0)	Z(7)	Z(2)
	Z(3)	Z(10)	Z(5)
	Z(6)	Z(1)	Z(8)
	Z(9)	Z(4)	Z(11)

which is actually

row ↓col→			
	X(0)	X(1)	X(2)
	X(3)	X(4)	X(5)
	X(6)	X(7)	X(8)
	X(9)	X(10)	X(11)

which is the original X, luckily. For this Knight's Tour on every alternate transformation we are going to get back the same .
Thus for order 2 we get,

X(0) = X(0);
X(1) = X(1);
X(2) = X(2);
X(3) = X(3);
X(4) = X(4);
X(5) = X(5);
X(6) = X(6);
X(7) = X(7);
X(8) = X(8);
X(9) = X(9);
X(10) = X(10);
X(11) = X(11);

which implies that all 12 bit numbers are chitrakavya numbers of order 2 for this Knight's tour because we have 12 independent simultaneous linear equations.

Let us take another Knight's tour possible. One of the other possible tour will be

0 7 2 3 10 5 6 11 4 9 8 1
The transformed Z will be

row ↓col→			
	Z(0)	Z(1)	Z(2)
	Z(3)	Z(4)	Z(5)
	Z(6)	Z(7)	Z(8)
	Z(9)	Z(10)	Z(11)

where
Z(0)=X(0)
Z(1)=X(7)
Z(2)=X(2)
Z(3)=X(3)
Z(4)=X(10)
Z(5)=X(5)
Z(6)=X(6)
Z(7)=X(11)
Z(8)=X(4)
Z(9)=X(9)
Z(10)=X(8)
Z(11)=X(1)

On taking the transform again on Z , we will get the transformed W where W will be given by

row ↓col→			
	W(0)	W(1)	W(2)
	W(3)	W(4)	W(5)
	W(6)	W(7)	W(8)
	W(9)	W(10)	W(11)

and
W(0)=Z(0)
W(1)=Z(7)
W(2)=Z(2)
W(3)=Z(3)
W(4)=Z(10)
W(5)=Z(5)
W(6)=Z(6)
W(7)=Z(11)
W(8)=Z(4)
W(9)=Z(9)
W(10)=Z(8)
W(11)=Z(1)

The W expressed in terms of elements of Z is given as

row ↓col→			
	Z(0)	Z(7)	Z(2)
	Z(3)	Z(10)	Z(5)
	Z(6)	Z(11)	Z(4)
	Z(9)	Z(8)	Z(1)

which when expressed in terms of elements of X is given as

row ↓col→			
	X(0)	X(11)	X(2)
	X(3)	X(8)	X(5)
	X(6)	X(1)	X(10)
	X(9)	X(4)	X(7)

For all the chitrakavya numbers of Order 2, the two matrices W and X must be same, which allows us to form the following set of equations,
X(0) = X(0);
X(1) = X(11);
X(2) = X(2);
X(3) = X(3);
X(4) = X(8);
X(5) = X(5);
X(6) = X(6);
X(7) = X(1);
X(8) = X(10);
X(9) = X(9);
X(10) = X(4);
X(11) = X(7);

removing the redundant equations, we get

X(0) = X(0);
X(1) = X(7)=X(11);
X(2) = X(2);
X(3) = X(3);

X(4) = X(8)=X(10);
X(5) = X(5);
X(6) = X(6);
X(9) = X(9);

Thus, we obtain 8 linearly independent equations. As in the previous section we follow the same steps to determine the chitrakavya numbers.

We now generate all numbers of 8 bits, i.e., from 0 to 2^8-1. We modify the equations as illustrated:

Y(0)=X(0) = X(0);
Y(1)=X(1) =
X(7)=X(11); Y(2)=X(2)
= X(2); Y(3)=X(3) =
X(3); Y(4)=X(4) =
X(8)=X(10); Y(5)=X(5)
= X(5); Y(6)=X(6) =
X(6); Y(7)=X(9) = X(9);

And we assign bits of any 8 bit number in the following fashion:
Y(i)= i th bit of the number

The number
X(11)X(10)X(9)X(8)X(7)X(6)X(5)X(4)X(3)X(2)X(1)X(0)
is thus
Y(1)Y(4)Y(7)Y(4)Y(1)Y(6)Y(5)Y(4)Y(3)Y(2)Y(1)Y(0)

and the total number of chitrakavya numbers we are going to get is 2^8 for this Knight's Tour.

Proceeding in this way we can obtain chitrakavya numbers for Order K by transforming the original X K times and then finding the linearly independent equations and then generating the solutions for these equations and then finding the corresponding Chitrakavya Numbers.

The important point to be noticed is that the largest order for a number of M bits is M.

V. RESULTS FOR 4 X 3

Knight's Tour0 7 2 3 10 5 6 11 4 9 8 1

Order	Number of Chitrakaya Numbers
1	256
2	256
3	4096
4	256
5	256
6	4096
7	256
8	256
9	4096
10	256
11	256
12	4096

Knight's Tour0 7 2 3 10 5 6 1 8 9 4 11

Order	Number of Chitrakaya Numbers
1	1024
2	4096
3	1024
4	4096
5	1024
6	4096
7	1024
8	4096
9	1024
10	4096
11	1024
12	4096

VI. CHITRAKAVYA NUMBERS BY N DIFFERENT KNIGHT'S TOUR

Up till now we have tried to find chitrakavya numbers for a particular Knight's Tour. It may happen that a particular number is a chitrakavya number for two or more possible Knight's Tour for the same Order. These numbers may be termed as magic chitrakavya numbers.

The steps required to find these magic chitrakavya numbers are:

1) **Find all the possible Knight's tour for M x N . Let these Knights' tour be denoted by K1, K2, K3, K4…Kn.**
2) **Let a chitrakavya number for Knight's Tour for order j be denoted by N(Ki,j).**
3) **If N(Ki,p) = N(Km,q) where p=q and Ki≠Km, we term these numbers as magic chitrakavya numbers.**
4) **Let us denote a number of M x N bits by Nmn. Then we list all Knights Tour for which Nmn is a chitrakavya number.**
5) **Let us denote all these Knight's Tour by K(m,i) where i denotes the order and m denotes the mth possible Knight's Tour M X N matrix.**
6) **Then we find all the Knight's Tour where K(m,i)=K(n,i). If we find one or more such**

solution, then the number is a magic chitrakavya number.

0 and the number represented by all bits set to 1 are always magic chitrakavya numbers.
For 8 x 4 the following numbers are magic chitrakavya numbers:

0
2147483647
2147483648
4294967295

The Tours were{0,9,15,6,8,17,23,30,24,22,31,25,16,14,7,1,10,9,26,28,21,27,29,20,13,4,2,11,18,12,5,3};
{0,9,15,6,8,17,23,30,24,22,31,25,16,14,7,1,10,3,5,11,2,4,13,20,29,27,18,12,21,28,26,19};
and{0,9,15,6,8,17,23,30,24,22,31,25,16,14,1,10,3,5,11,2,4,13,20,29,27,18,12,21,28,26,19};

Thus, we find only three magic chitrakavya numbers for 8 x 4 these possible Knight's tour.

VII Regular Expressions

We replaced the Boolean expression with regular expressions on a 8 X 4 chessboard. The expressions were permuted based on knights tour for a 8 X 4 chessboard. The total number of knights tours for the given chessboard are 62176
. The main aim was to find if the expressions formed on permutations are syntactically and grammatically correct. What we found was that , the number of syntactically correct expression Depends on how the expression is placed on the chessboard squares. The same expression when placed differently on the squares gives different permutations that can be found from the same set of variables.