Abelian Surfaces with Quaternionic Multiplication

Prof. Vijay Patankar (JNU, Delhi)

Let A be an absolutely simple Abelian surface defined over a number field K with multiplication by an indefinite quaternion division algebra over the field of rational numbers. Let v be a finite prime of K of good reduction for A. Let k(v) be the residue field of K at v. Then, a well-known result says that the reduction of A modulo v is isogenous over a some finite extension of k(v) to the square of some elliptic curve E(v) defined over k(v). We refine this result to prove that over a large enough number field K, such an A has good reduction everywhere, and that for any finite place v of K, the reduction of A modulo v is isogenous over k(v) to the square of E(v) defined over k(v).

This provides an example of an L-function which is not a square of a global L-function but whose Euler products at all finite places are in fact squares.